Certified Spectral Boundary from Heat-Kernel Budgets and Entropic Transport in the Einstein-Locked OT/GKSL Framework

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We develop a certified spectral-boundary theory inside the Einstein-locked optimal-transport/Gorini–Kossakowski–Sudarshan–Lindblad (OT/GKSL) framework, building on the GKSL theory of quantum dynamical semigroups [1, 2] and on the certified-domain OT/GKSL architecture introduced in Refs. [12–14]. The native level is not a primitive spacetime geometry, but an open-system dynamics of density operators on an ambient internal support. In the detailed-balance sector, this dynamics carries a quantum optimal-transport gradient-flow structure for relative entropy, in the sense of the Carlen–Maas formulation of reversible quantum Markov semigroups [4]. Classical spacetime geometry appears only as a certified readout on operational windows where temporal records, coframe nondegeneracy, and OT-to-readout bridge control remain jointly admissible.

The purpose of this manuscript is to graft heat-kernel spectral geometry [7–9] onto this native/readout architecture without converting it into a Sakharov-type induced-gravity argument [11] and without relaxing the Einstein lock. We distinguish the native heat trace, used to control finite spectral resources,

$$K_\rho^{\mathrm{nat}}(s) = \mathrm{Tr}_{\mathcal{H}_\gamma} \exp\left[-sD_{\mathrm{int}}(\rho)^2\right],$$

from the readout heat trace, used only after certification to organize Seeley–DeWitt slots. The cutoff projector

$$P_{\Lambda_\star}(\rho) = \mathbf{1}_{[0,\Lambda_\star]}(|D_{\mathrm{int}}(\rho)|)$$

defines the certified effective support and its dimension

$$d_{\mathrm{eff}}(\Lambda_\star;\rho) = \mathrm{Tr}\, P_{\Lambda_\star}(\rho).$$

The first result is a heat-kernel control of the finite spectral budget,

$$\log d_{\mathrm{eff}}(\Lambda_\star;\rho) \leq \inf_{s>0} \left[s\Lambda_\star^2 + \log K_\rho^{\mathrm{nat}}(s)\right].$$

The second result promotes the spectral gap at the cutoff to a certification margin,

$$m_{\mathrm{spec}}(\rho) = \mathrm{dist}(\Lambda_\star, \mathrm{Spec}\,|D_{\mathrm{int}}(\rho)|)\,,$$

with an associated coercive certification charge diverging as $m_{\mathrm{spec}} \to 0$. The third result is an entropic transport estimate: along the detailed-balance OT/GKSL flow, the variation of the heat-kernel spectral budget is bounded by the entropic length of the trajectory,

$$\left|\Delta \log K^{\mathrm{nat}}(s)\right| \leq L_s \int_{\xi_1}^{\xi_2} \sqrt{\sigma(\rho_\xi)}\, d\xi.$$

Combining these ingredients gives a certified spectral-boundary theorem. If the total readout charge, including the spectral-gap charge, exceeds the entropic plus heat-kernel spectral budget, then certified spacetime readout is obstructed, although the native OT/GKSL dynamics need not fail. Finally, we formulate the Einstein-locked slot-placement rule: the readout heat-kernel expansion may organize source-side, response, holonomic, vacuum-like, interface, and certification slots, but it cannot generate a state-dependent prefactor multiplying $R[g_{\mathrm{ro}}]$. Thus spectral geometry enters the OT/GKSL framework as a finite-resource and slot-control layer, not as an induced-gravity mechanism.

I. PROBLEM AND STRUCTURAL AIM

The aim of this manuscript is to construct a spectral-geometric extension of the Einstein-locked OT/GKSL framework that is mathematically nontrivial but structurally conservative. The paper is conditional on the certified-domain OT/GKSL architecture developed in the companion works [12–15]: native open-system dynamics, certified readout windows, the Einstein lock, and the finite readout-budget principle are taken as part of the background framework. The present contribution does not rederive those ingredients. It adds a spectral-geometric control layer to them.

The extension must satisfy three requirements.

First, it must preserve the native/readout hierarchy introduced in Refs. [12, 14]. The native theory is an OT/GKSL dynamics of density operators and is not formulated as a primitive spacetime field theory. Classical spacetime geometry appears only as a certified readout, on domains where temporal records, coframe nondegeneracy, and bridge control remain jointly admissible.

Second, the extension must preserve the Einstein lock, namely the placement rule by which the certified readout metric sector keeps a universal Einstein–Hilbert kinetic block while preparation-dependent effects are assigned to source, response, bridge, interface, or certification channels [15]. The certified readout metric sector has the universal Einstein–Hilbert kinetic block with constant G_0. Readable state dependence is not allowed to multiply $R[g_{\rm ro}]$. Any preparation-sensitive contribution must be placed in source, response, holonomic, interface, or certification sectors.

Third, the extension must yield more than a formal analogy between GKSL diffusion [1, 2, 5, 6] and heat-kernel diffusion [7, 9]. The heat-kernel parameter s, the native OT/GKSL parameter ξ, the entropic ordering $t_{\rm ent}$, and the readout time X^0 are distinct quantities. The heat kernel is used here as a spectral control functional for finite readout resources, not as a physical time evolution and not as an induced-gravity mechanism.

The central claim is therefore the following. The internal heat kernel of the native spectral operator controls the finite spectral budget required for certified spacetime readout. When the spectral operator depends on the density state, this budget is transported by the native entropic OT/GKSL flow. Moreover, stability of the cutoff support requires a spectral-gap margin, whose collapse defines a genuine spectral boundary of certified readout.

Physically, this means that the loss of classical spacetime readability can be driven by failure of spectral support stability, not by a breakdown of the native open-system dynamics itself. The native density-matrix evolution may continue to be meaningful, while the finite spectral resources needed to support a certified classical readout cease to be available or cease to be stable.

This claim is deliberately weaker than deriving spacetime from spectral geometry, but stronger than a bookkeeping analogy. It produces an obstruction criterion for certified readout and a dynamical control estimate for spectral-resource variation along the native entropy-gradient flow.

II. NATIVE OT/GKSL LAYER AND CERTIFIED READOUT HIERARCHY

Let $\mathcal{D}^\circ(\mathcal{H})$ denote the manifold of faithful density operators on the relevant native support. The native dynamics is a GKSL evolution [1, 2, 5, 6]

$$\frac{d\rho_\xi}{d\xi} = \mathcal{L}(\rho_\xi), \tag{II.1}$$

where ξ is the native flow parameter. It is not a spacetime coordinate and it is not assumed to coincide with any operational clock.

In the detailed-balance sector, relative to a faithful stationary state π, the flow admits the quantum optimal-transport gradient-flow form [4]

$$\frac{d\rho_\xi}{d\xi} = -\operatorname{grad}_{OT} D(\rho_\xi \| \pi), \tag{II.2}$$

where

$$D(\rho\|\pi) = \mathrm{Tr}[\rho(\log\rho - \log\pi)] \tag{II.3}$$

is the relative entropy. The associated entropy production, in the sense of entropy production for quantum dynamical semigroups [3], is

$$\sigma(\rho_\xi) := -\frac{d}{d\xi} D(\rho_\xi\|\pi) = \left\| \frac{d\rho_\xi}{d\xi} \right\|^2_{g_{OT}} \geq 0. \tag{II.4}$$

The native entropic ordering is defined by

$$\frac{dt_{\rm ent}}{d\xi} = \sigma(\rho_\xi). \tag{II.5}$$

This quantity is internal to the state-space dynamics. It is not yet a classical tick and not yet the readout time coordinate.

Certified spacetime readout, as developed in the certified-readout sector of the framework [13, 14], is asserted only on a certified window

$$W_{\rm st} \subseteq W_{\rm acc}, \tag{II.6}$$

where temporal readability, coframe nondegeneracy, and bridge control are simultaneously maintained. The present work introduces a further spectral subwindow

$$W_{\rm spec} \subseteq W_{\rm st}, \tag{II.7}$$

on which the cutoff spectral support remains stable and the heat-kernel budget is controlled.

III. AMBIENT INTERNAL SPECTRAL OPERATOR AND CUTOFF SUPPORT

A key point is that the internal spectral operator is defined before the certified cutoff support is selected. This avoids a circular definition of the effective dimension.

Assumption 1 (Ambient internal support)**.** *There exists an ambient internal Hilbert space $\mathcal{H}_\gamma$ carrying the internal spinorial/spectral degrees of freedom of the native sector.*

Assumption 2 (Regular internal Dirac-type operator)**.** *On a spectral working domain, there exists a self-adjoint internal Dirac-type operator*

$$D_{\rm int}(\rho) : \mathcal{H}_\gamma \to \mathcal{H}_\gamma. \tag{III.1}$$

We define the positive self-adjoint operator

$$H_\rho^{\rm nat} := D_{\rm int}(\rho)^2 \geq 0. \tag{III.2}$$

We assume that $e^{-sH_\rho^{\rm nat}}$ is trace-class for all $s > 0$ in the working range and that the cutoff spectral projector introduced below has finite trace. In the finite-dimensional case these trace assumptions are automatic.

Definition 1 (Cutoff projector and effective support)**.** *For a cutoff $\Lambda_\star > 0$, define*

$$P_{\Lambda_\star}(\rho) = \mathbf{1}_{[0,\Lambda_\star]}(|D_{\rm int}(\rho)|) = \mathbf{1}_{[0,\Lambda_\star^2]}\left(H_\rho^{\rm nat}\right). \tag{III.3}$$

The certified effective support is

$$\mathcal{H}_{\rm eff}(\Lambda_\star;\rho) = P_{\Lambda_\star}(\rho)\mathcal{H}_\gamma, \tag{III.4}$$

with effective spectral dimension

$$d_{\rm eff}(\Lambda_\star;\rho) = \operatorname{Tr}_{\mathcal{H}_\gamma} P_{\Lambda_\star}(\rho). \tag{III.5}$$

Assumption 3 (Nonempty effective support)**.** *On the spectral working window, the cutoff selects a nonempty effective support,*

$$d_{\rm eff}(\Lambda_\star;\rho) \geq 1. \tag{III.6}$$

This makes $\log d_{\rm eff}$ well-defined.

Definition 2 (Native heat trace)**.** *The native heat trace is*

$$K_\rho^{\rm nat}(s) = \operatorname{Tr}_{\mathcal{H}_\gamma} e^{-sH_\rho^{\rm nat}}, \qquad s > 0. \tag{III.7}$$

The parameter s is a spectral heat parameter and is distinct from ξ, $t_{\rm ent}$, and X^0.

IV. NATIVE HEAT-KERNEL BOUND ON THE EFFECTIVE DIMENSION

The first result is a spectral counting estimate. It belongs to the standard family of heat-kernel counting bounds used in spectral geometry [7–9]. It is elementary as a spectral inequality, but its role here is structural: it converts the finite number of certified modes into a heat-kernel-controlled spectral resource.

Lemma 1 (Native heat-kernel counting bound)**.** *Let $H_\rho^{\rm nat} \geq 0$ and $K_\rho^{\rm nat}(s) = \operatorname{Tr} e^{-sH_\rho^{\rm nat}}$. Then, for every $s > 0$,*

$$d_{\rm eff}(\Lambda_\star;\rho) \leq e^{s\Lambda_\star^2} K_\rho^{\rm nat}(s). \tag{IV.1}$$

Proof. For each eigenvalue $\lambda \geq 0$ of H_ρ^{nat},

$$\mathbf{1}_{[0,\Lambda_\star^2]}(\lambda) \leq e^{s\Lambda_\star^2} e^{-s\lambda}. \tag{IV.2}$$

Functional calculus gives

$$\mathbf{1}_{[0,\Lambda_\star^2]}(H_\rho^{\mathrm{nat}}) \leq e^{s\Lambda_\star^2} e^{-sH_\rho^{\mathrm{nat}}}. \tag{IV.3}$$

Taking the trace yields Eq. (IV.1). □

Corollary 1 (Logarithmic spectral-resource bound)**.** *For every $s > 0$,*

$$\log d_{\mathrm{eff}}(\Lambda_\star; \rho) \leq s\Lambda_\star^2 + \log K_\rho^{\mathrm{nat}}(s). \tag{IV.4}$$

Consequently,

$$\log d_{\mathrm{eff}}(\Lambda_\star; \rho) \leq \inf_{s>0} \left[s\Lambda_\star^2 + \log K_\rho^{\mathrm{nat}}(s) \right]. \tag{IV.5}$$

Remark 1. *When an exact heat trace is known, the infimum may be taken over the full working range $s > 0$. When only an asymptotic Seeley–DeWitt expansion is used, the corresponding estimate must be restricted to the range of validity of that expansion, for instance $0 < s < s_0$.*

V. SPECTRAL GAP MARGIN AND CERTIFIED SPECTRAL CHARGE

The cutoff projector is stable only if the cutoff does not sit on the spectrum of $|D_{\mathrm{int}}(\rho)|$. This motivates a new certification margin.

Definition 3 (Spectral gap margin)**.** *The spectral cutoff margin is*

$$m_{\mathrm{spec}}(\rho) = \mathrm{dist}(\Lambda_\star, \mathrm{Spec}\,|D_{\mathrm{int}}(\rho)|). \tag{V.1}$$

For a fixed margin $\delta_\Lambda > 0$, define the certified spectral subwindow

$$W_{\mathrm{spec}}(\delta_\Lambda) = \{\rho \in W_{\mathrm{st}} : \; m_{\mathrm{spec}}(\rho) \geq \delta_\Lambda\}. \tag{V.2}$$

The spectral boundary is approached when $m_{\mathrm{spec}}(\rho) \to 0$.

Definition 4 (Certified spectral charge)**.** *Let $\Phi_{\mathrm{spec}} : (0, \infty) \to (0, \infty)$ be a coercive spectral cost function satisfying*

$$\Phi_{\mathrm{spec}}(m) \to +\infty \qquad \text{as} \qquad m \downarrow 0. \tag{V.3}$$

The spectral certification charge is

$$C_{\mathrm{spec}}(\rho) = \Phi_{\mathrm{spec}}(m_{\mathrm{spec}}(\rho)). \tag{V.4}$$

A minimal proxy used for explicit estimates is

$$C_{\mathrm{spec}}(\rho) = \frac{\kappa_{\mathrm{spec}}}{m_{\mathrm{spec}}(\rho)}, \qquad \kappa_{\mathrm{spec}} > 0. \tag{V.5}$$

Lemma 2 (Spectral-margin collapse)**.** *If $m_{\mathrm{spec}}(\rho_n) \to 0$ along a sequence $\rho_n \in W_{\mathrm{st}}$, then*

$$C_{\mathrm{spec}}(\rho_n) \to +\infty. \tag{V.6}$$

Therefore spectral cutoff instability cannot be certified at finite spectral charge.

Proof. The claim follows from the coercivity of Φ_{spec} at the origin. For the minimal proxy, it follows immediately from Eq. (V.5). □

Remark 2. *The spectral gap margin plays the same structural role as temporal, geometric, and bridge margins in the certified-readout architecture. Its collapse does not imply a failure of the native OT/GKSL dynamics. It implies a loss of certified stability of the cutoff support used by the readout.*

VI. ENTROPIC TRANSPORT ESTIMATE FOR HEAT-KERNEL BUDGETS

The previous heat-kernel bound is static. We now show that, when D_{int} depends on ρ, the variation of the heat-kernel spectral resource is controlled by the entropic length of the OT/GKSL trajectory.

Let

$$\mathcal{I}_s(\rho) = \log K^{\mathrm{nat}}_{\rho}(s). \tag{VI.1}$$

Assume $\mathcal{I}_s$ is differentiable on $W_{\mathrm{spec}}(\delta_\Lambda)$ and has an OT-gradient $\nabla_{OT}\mathcal{I}_s$.

Definition 5 (Spectral-response Lipschitz constant)**.** *For fixed $s > 0$, define the certified-window spectral-response constant*

$$L_s(W_{\mathrm{spec}}) := \sup_{\rho\in W_{\mathrm{spec}}(\delta_\Lambda)} \left\|\nabla_{OT} \log K^{\mathrm{nat}}_{\rho}(s)\right\|_{OT}, \tag{VI.2}$$

whenever the supremum is finite.

The quantity L_s measures the spectral susceptibility of the native heat trace under OT-displacements of the density state. It is not assumed to be universal. It is a certified-window constant: it is meaningful only on a domain where the spectral cutoff support is stable, the OT metric is controlled, and the heat trace responds regularly to changes in preparation. A large L_s means that small entropic displacements can induce large spectral-resource drift; a small L_s means that the heat-kernel budget is robust along the native trajectory.

Lemma 3 (Pointwise entropic transport estimate)**.** *Along a detailed-balance OT/GKSL trajectory,*

$$\dot\rho_\xi = -\operatorname{grad}_{OT} D(\rho_\xi\|\pi), \tag{VI.3}$$

one has

$$\left|\frac{d}{d\xi}\log K^{\mathrm{nat}}_{\rho_\xi}(s)\right| \le \left\|\nabla_{OT} \log K^{\mathrm{nat}}_{\rho_\xi}(s)\right\|_{OT} \sqrt{\sigma(\rho_\xi)}. \tag{VI.4}$$

Proof. By the chain rule and the OT-gradient representation,

$$\begin{aligned}\frac{d}{d\xi}\mathcal{I}_s(\rho_\xi) &= d\mathcal{I}_s[\dot\rho_\xi] \qquad &\text{(VI.5)}\\ &= -g_{OT}(\nabla_{OT}\mathcal{I}_s, \operatorname{grad}_{OT} D)\,. &\text{(VI.6)}\end{aligned}$$

Using Cauchy–Schwarz in the OT metric gives

$$\left|\frac{d}{d\xi}\mathcal{I}_s(\rho_\xi)\right| \le \|\nabla_{OT}\mathcal{I}_s\|_{OT}\,\|\operatorname{grad}_{OT} D\|_{OT}. \tag{VI.7}$$

By Eq. (II.4), $\|\operatorname{grad}_{OT} D\|^2_{OT} = \sigma(\rho_\xi)$, yielding Eq. (VI.4). □

Theorem 1 (Integrated entropic heat-budget control)**.** *Assume that a trajectory segment $[\xi_1, \xi_2]$ remains inside $W_{\mathrm{spec}}(\delta_\Lambda)$ and that $L_s(W_{\mathrm{spec}}) < \infty$. Then*

$$\left|\log K^{\mathrm{nat}}_{\rho_{\xi_2}}(s) - \log K^{\mathrm{nat}}_{\rho_{\xi_1}}(s)\right| \le L_s \int_{\xi_1}^{\xi_2} \sqrt{\sigma(\rho_\xi)}\, d\xi. \tag{VI.8}$$

Moreover,

$$\left|\Delta \log K^{\mathrm{nat}}(s)\right| \le L_s\sqrt{\xi_2-\xi_1}\sqrt{D(\rho_{\xi_1}\|\pi) - D(\rho_{\xi_2}\|\pi)}. \tag{VI.9}$$

Proof. Integrating Eq. (VI.4) gives Eq. (VI.8). For Eq. (VI.9), apply Cauchy–Schwarz:

$$\int_{\xi_1}^{\xi_2}\sqrt{\sigma(\rho_\xi)}\,d\xi \le \sqrt{\xi_2-\xi_1}\left(\int_{\xi_1}^{\xi_2}\sigma(\rho_\xi)\,d\xi\right)^{1/2}. \tag{VI.10}$$

Using

$$\int_{\xi_1}^{\xi_2}\sigma(\rho_\xi)\,d\xi = D(\rho_{\xi_1}\|\pi) - D(\rho_{\xi_2}\|\pi) \tag{VI.11}$$

completes the proof. □

VII. CERTIFIED SPECTRAL BOUNDARY THEOREM

We now combine the heat-kernel spectral bound, the spectral-gap certification charge, and the entropic transport estimate.

The certified spacetime-readout budget is taken to contain an entropic and a spectral contribution:

$$B_{\mathrm{st}} = \alpha_{\mathrm{ent}} \Delta t_{\mathrm{ent}} + \alpha_d \log d_{\mathrm{eff}}(\Lambda_\star; \rho). \tag{VII.1}$$

The total certification charge includes the ordinary temporal, geometric, and bridge charges, together with the spectral charge:

$$C_{\mathrm{tot}} = C_\tau + C_{\mathrm{geo}} + C_{\mathrm{br}} + C_{\mathrm{spec}}. \tag{VII.2}$$

Theorem 2 (Certified spectral boundary)**.** *On $W_{\mathrm{spec}}(\delta_\Lambda)$, the certified readout budget obeys*

$$B_{\mathrm{st}} \le \alpha_{\mathrm{ent}} \Delta t_{\mathrm{ent}} + \alpha_d \inf_{s>0} \left[s\Lambda_\star^2 + \log K_\rho^{\mathrm{nat}}(s) \right]. \tag{VII.3}$$

Consequently, if

$$C_{\mathrm{tot}} > \alpha_{\mathrm{ent}} \Delta t_{\mathrm{ent}} + \alpha_d \inf_{s>0} \left[s\Lambda_\star^2 + \log K_\rho^{\mathrm{nat}}(s) \right], \tag{VII.4}$$

then certified spacetime readout is obstructed.

Furthermore, if $m_{\mathrm{spec}}(\rho) \to 0$, then $C_{\mathrm{spec}}(\rho) \to +\infty$, and certified readout fails at finite available budget.

Proof. The bound Eq. (VII.3) follows by inserting Eq. (IV.5) into Eq. (VII.1). Certified readout requires

$$C_{\mathrm{tot}} \le B_{\mathrm{st}}. \tag{VII.5}$$

If Eq. (VII.4) holds, then C_{tot} exceeds an upper bound on B_{st}, hence $C_{\mathrm{tot}} > B_{\mathrm{st}}$, and the certified readout condition fails. The final statement follows from the coercivity of C_{spec} as $m_{\mathrm{spec}} \to 0$. □

Remark 3. *This is an obstruction theorem. It does not claim that satisfying the inequality in the opposite direction is sufficient for readout certification. Temporal, geometric, bridge, and spectral margins must all remain jointly admissible.*

Corollary 2 (Dynamical persistence of the heat-kernel budget margin)**.** *Fix $s > 0$ and define the heat-kernel upper budget functional*

$$\mathcal{U}_s(\xi) := \alpha_{\mathrm{ent}} \Delta t_{\mathrm{ent}}(\xi) + \alpha_d \left[s\Lambda_\star^2 + \log K_{\rho_\xi}^{\mathrm{nat}}(s) \right]. \tag{VII.6}$$

Suppose that on $[\xi_1, \xi_2]$ the trajectory remains in $W_{\mathrm{spec}}(\delta_\Lambda)$, $L_s < \infty$, and the non-spectral charge variation is bounded by

$$|\Delta C_{\mathrm{ns}}| \le M_C, \qquad C_{\mathrm{ns}} := C_\tau + C_{\mathrm{geo}} + C_{\mathrm{br}}. \tag{VII.7}$$

If at ξ_1 the heat-kernel margin satisfies

$$\mathcal{U}_s(\xi_1) - C_{\mathrm{tot}}(\xi_1) > \alpha_d L_s \int_{\xi_1}^{\xi_2} \sqrt{\sigma(\rho_\xi)}\, d\xi + M_C, \tag{VII.8}$$

and if C_{spec} does not increase by more than the remaining margin, then the heat-kernel obstruction condition cannot be triggered on $[\xi_1, \xi_2]$ solely by heat-trace variation and bounded non-spectral charge drift.

Proof. The possible decrease of the heat-kernel part of $\mathcal{U}_s$ over the interval is bounded by Eq. (VI.8) multiplied by α_d. The non-spectral charge drift is bounded by M_C. Under condition Eq. (VII.8), these effects cannot exhaust the initial heat-kernel margin. The remaining qualification concerns the explicitly spectral charge, whose divergence is controlled separately by m_{spec}. □

Remark 4. *The corollary is not a sufficiency theorem for full spacetime readout. It is a persistence statement for the absence of the heat-kernel spectral obstruction under controlled entropic length and bounded charge drift.*

VIII. READOUT HEAT KERNEL AND EINSTEIN-LOCKED SLOT PLACEMENT

The native heat kernel $K_\rho^{\mathrm{nat}}(s)$ controls spectral resources before readout. A distinct heat kernel may be introduced only after certification of a readout geometry.

On W_{st}, suppose a readout Dirac-type operator exists:

$$D_{\mathrm{ro}}(\rho, g_{\mathrm{ro}}, A, Y), \tag{VIII.1}$$

where g_{ro} is the certified readout metric, A denotes admissible gauge/readout connections, and Y denotes source-side endomorphism data. The readout heat trace is

$$K_\rho^{\mathrm{ro}}(s) = \mathrm{Tr}\, e^{-sD_{\mathrm{ro}}(\rho, g_{\mathrm{ro}}, A, Y)^2}. \tag{VIII.2}$$

When D_{ro}^2 is of generalized Laplace type, one has a Seeley–DeWitt expansion

$$K_\rho^{\mathrm{ro}}(s) \sim \sum_{k\geq 0} s^{(k-d)/2} a_k^{\mathrm{ro}}(\rho, g_{\mathrm{ro}}, A, Y), \qquad s \downarrow 0. \tag{VIII.3}$$

Proposition 1 (Einstein-locked spectral placement rule)**.** *The readout heat-kernel coefficients, understood in the standard Seeley–DeWitt sense [7–9], may organize admissible source-side, response, gauge, holonomic, vacuum-like, interface, and certification slots. They cannot generate a readable state-dependent prefactor multiplying the Ricci scalar in the gravitational kinetic block.*

Thus,

$$a_0^{\mathrm{ro}} \longrightarrow \textit{vacuum-like source-side slot}, \tag{VIII.4}$$
$$a_2^{\mathrm{ro}} R[g_{\mathrm{ro}}] \longrightarrow \textit{universal Einstein–Hilbert block}, \tag{VIII.5}$$
$$a_2^{\mathrm{ro}} \textit{ endomorphism terms} \longrightarrow \textit{constitutive source/response slots}, \tag{VIII.6}$$
$$a_4^{\mathrm{ro}}, a_6^{\mathrm{ro}}, \ldots \longrightarrow \textit{gauge, response, holonomic, interface, or certification slots.} \tag{VIII.7}$$

In particular, a term $f(\rho)R[g_{\mathrm{ro}}]$ is inadmissible as a kinetic gravitational term unless $f(\rho)$ is constant on the certified readout sector.

Proof. This is a consistency statement following from the Einstein lock. The certified readout gravitational sector has a universal Einstein–Hilbert kinetic block with constant G_0. Therefore readable preparation dependence cannot be assigned to the coefficient of $R[g_{\mathrm{ro}}]$. Such dependence must be projected to source, response, interface, holonomic, or certification structures. □

Remark 5. *This proposition is not an independent derivation from spectral geometry alone. It is an Einstein-locked consistency rule for placing heat-kernel slots within the OT/GKSL readout architecture. It is precisely the point at which the present construction differs from a Sakharov-like or spectral-action derivation of gravity [10, 11].*

IX. VACUUM-LIKE RESIDUAL SLOT UNDER SPECTRAL-BUDGET CONTROL

As an application to the homogeneous and reduced sectors of the framework [16, 17], suppose a certified reduced constitutive–holonomic sector has a stationary branch with residual energy

$$E_{\mathrm{vac}} = U_{\mathrm{eff}}(r_\star). \tag{IX.1}$$

In the homogeneous Einstein-locked readout sector, this residual may populate the vacuum-like slot through

$$\rho_\Lambda^{\mathrm{slot}} = K_{\mathrm{match}}\, d_{\mathrm{eff}}(\Lambda_\star; \rho)\, E_{\mathrm{vac}}. \tag{IX.2}$$

Corollary 3 (Heat-kernel control of the vacuum-like slot)**.** *For every $s > 0$,*

$$|\rho_\Lambda^{\mathrm{slot}}| \leq |K_{\mathrm{match}} E_{\mathrm{vac}}| e^{s\Lambda_\star^2} K_\rho^{\mathrm{nat}}(s). \tag{IX.3}$$

Consequently,

$$|\rho_\Lambda^{\mathrm{slot}}| \leq |K_{\mathrm{match}} E_{\mathrm{vac}}| \inf_{s>0} \left[e^{s\Lambda_\star^2} K_\rho^{\mathrm{nat}}(s) \right]. \tag{IX.4}$$

Proof. The result follows from the heat-kernel counting bound applied to d_{eff}. □

Remark 6. *The vacuum-like contribution is controlled by the native spectral budget. It is not introduced as a primitive cosmological constant and it does not modify the Einstein–Hilbert kinetic block.*

X. PHYSICAL INTERPRETATION

The certified spectral boundary has a simple physical interpretation. The native OT/GKSL dynamics supplies a trajectory of density states. The internal spectral operator assigns to each such state a spectrum, and the cutoff selects the finite portion of that spectrum available to the certified readout. The heat trace controls how many spectral modes are available below the cutoff, while the gap margin controls whether the cutoff selection is stable.

Thus the spectral sector contributes to readout in two logically distinct ways. First, it supplies capacity: the number of effective modes below $\Lambda_\star$ contributes to the finite information budget through $\log d_{\mathrm{eff}}$. Second, it supplies stability: the cutoff must remain separated from the spectrum by a nonzero margin. Capacity without stability is not enough; a large number of modes is not certifiable if the projector selecting them is about to jump. Stability without capacity is also not enough; a stable but too small support may fail to provide the resources required for temporal, geometric, and bridge readout.

The heat-kernel bound controls the capacity side. The spectral gap charge controls the stability side. The entropic transport estimate controls the dynamical side: it states that along a detailed-balance OT/GKSL trajectory, the logarithmic heat budget cannot drift faster than permitted by the entropic length of the path and the spectral-response constant L_s.

This differs from a conventional induced-gravity interpretation. In the present construction, the heat kernel does not create the Einstein–Hilbert kinetic term. The Einstein–Hilbert block is already locked in the certified readout sector. Spectral geometry instead tells us whether the finite spectral resources required to sustain that readout remain available, stable, and dynamically controlled.

In this sense, the result is best understood as a theory of failure modes for classical readability. A spectral boundary is reached when the certified support cannot be maintained at finite charge. The native open-system evolution may continue, but the classical spacetime description loses its certified license.

XI. DISCUSSION

The construction above produces a spectral-geometric layer inside the Einstein-locked OT/GKSL framework without changing the framework's native/readout architecture. The result is conditional rather than standalone: it assumes the certified-domain OT/GKSL framework, including the existence of readout windows, the Einstein lock, and finite budget constraints [12–15]. The role of the heat kernel is not to induce the gravitational kinetic block. It is to control finite spectral resources, certification margins, and admissible readout slots.

The main difference from a conventional spectral-action or Sakharov-type route is the order of interpretation. In standard spectral-geometric settings, one often uses the heat-kernel expansion to obtain geometric terms in an effective action. Here the certified readout metric is already subject to the Einstein lock. Therefore the readout heat kernel may organize admissible slots, but it cannot make the gravitational kinetic coefficient state-dependent.

The principal new object is the certified spectral boundary. It has two complementary aspects. First, the heat trace bounds the number of certified spectral modes below the cutoff. Second, the cutoff gap defines a certification margin. If the gap collapses, the support selected by the cutoff becomes unstable and the spectral certification charge diverges. This gives a precise sense in which spectral instability can obstruct classical spacetime readout without implying a failure of the native OT/GKSL dynamics.

The entropic transport estimate gives the dynamical part of the result. If the internal spectral operator depends on the density state, heat-kernel resources vary along the native OT/GKSL trajectory. The variation of the logarithmic heat budget is bounded by the entropic length of the trajectory. Thus spectral-resource drift is not arbitrary: it is controlled by entropy production in the detailed-balance OT/GKSL sector.

Finally, the vacuum-like corollary shows how reduced-sector residual energy, if present, remains under heat-kernel spectral-budget control when lifted into a homogeneous Einstein-locked readout slot. This is a corollary of the budget theory, not a new primitive cosmological constant.

Several limitations should be kept explicit. The certified boundary theorem is one-sided; it gives an obstruction, not a full certification criterion. The constant L_s and the spectral cost function Φ_{spec} are certified-window quantities whose concrete values require a more explicit model of $D_{\mathrm{int}}(\rho)$. Finally, the present paper controls heat-trace drift but only models spectral-gap collapse through a coercive charge. A future strengthening should derive OT-transport estimates for m_{spec} itself.

XII. CONCLUSION

We have constructed a certified spectral-boundary layer for the Einstein-locked OT/GKSL framework. The construction preserves the native/readout hierarchy, keeps the Einstein–Hilbert kinetic sector locked, and uses heat-kernel spectral geometry as a finite-resource control tool rather than as an induced-gravity mechanism.

The main results are threefold. First, the native heat trace bounds the effective number of certified spectral modes:

$$\log d_{\rm eff}(\Lambda_\star;\rho) \le \inf_{s>0}\left[s\Lambda_\star^2 + \log K_\rho^{\rm nat}(s)\right]. \tag{XII.1}$$

Second, the spectral gap at the cutoff defines a new certification margin,

$$m_{\rm spec}(\rho) = {\rm dist}(\Lambda_\star, {\rm Spec}\,|D_{\rm int}(\rho)|)\,, \tag{XII.2}$$

whose collapse forces the spectral certification charge to diverge. Third, when the internal spectral operator depends on the density state, the variation of the heat-kernel budget is controlled by the entropic length of the detailed-balance OT/GKSL trajectory,

$$\left|\Delta \log K^{\rm nat}(s)\right| \le L_s \int \sqrt{\sigma(\rho_\xi)}\, d\xi. \tag{XII.3}$$

Together these statements yield a certified spectral-boundary theorem: if the total readout charge, including the spectral charge, exceeds the entropic plus heat-kernel spectral budget, certified spacetime readout is obstructed, while the native OT/GKSL dynamics need not fail. The physical content is that classical spacetime readout can fail because the spectral resources required to certify it become insufficient or unstable, even though the underlying open-system state dynamics remains well-defined.

The readout heat-kernel expansion may still organize Seeley–DeWitt slots after certification, but the Einstein lock forbids any state-dependent prefactor of $R[g_{\rm ro}]$. Thus spectral geometry enters the OT/GKSL framework as a control theory of finite spectral resources, certification margins, and source/readout slots.

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